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Cancellation of unphysical gauge and ghost degrees of freedom in backreaction

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ABSTRACT

We study the U(1) Higgs model in spacetime-dependent background fields (a background metric and a background scalar field). Particle creation can occur because of the time-dependence of these background fields. In gauge theories, there is a unphysical sector and consequently unphysical particles may be produced. However, it is shown that produced unphysical particles have no contribution to backreaction to background fields.

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I Introduction

Inflationary universe scenario is intended to solve some of fundamental problems in the standard cosmology such as the horizon, flatness and primordial monopole problems [1][2] [3]. It introduces an exponentially expanding era due to the vacuum energy of the inflaton field. Gigantic order of expansion can solve two of the above problems ; the horizon and primordial monopole problems. It is intuitively expected that immediately after inflation, the vacuum energy is converted into radiation energy, and that the Friedmann expansion takes over the de Sitter expansion. And at this era, the huge amount of entropy is expected to be produced, which solves the flatness problem.

According to the new inflationary universe scenario [4] [5] , GUT phase transition is of second order. Not only a c-number background metric but also a c-number background Higgs field are time-dependent. Relations between these c-number background fields and quantum fluctuations have been studied for many years. In the context of the thermalization after inflation, several authors studied the effective evolution equation of a c-number background Higgs field [6][7][8][9]. However, several important problems are still left unsolved.

In this paper, we discuss some aspects of the consistency of quantum field theory when there are spacetime-dependent c-number background fields. In general, particle production can occur in spacetime-dependent background. Unphysical particles in gauge theories are no exception. However, we shall show at the 1-loop level that unphysical particles do not contribute to backreaction in spite of their condensation. We prove this fact by using the U(1) Higgs model, but extension to more complicated models (e.g. SU(5) GUT model) is straightforward.

This paper is organized as follows: In sect. II we explain our U(1) Higgs model. In particular we present a gauge fixing condition which is a generalization of the familiar

R_ξ gauge to spacetime-dependent background. This plays a crucial role in the following discussion. In sect. III a free part of the Lagrangian density is diagonalized, and mode expansion of each component is discussed in sect. IV. In sect. V we point out that the discrepancy between the in-vacuum and the out-vacuum lead to the condensation of physical and even unphysical particles. However, we can take BRST-invariant vacuum states on a certain condition. In sect. VI we show that the condensation of unphysical particles do not contribute to backreaction in the background field equations of a physical state. Sect. VII is for conclusion.

II Lagrangian

We start from the standard Lagrangian density for the U(1) Higgs model with a non-minimal coupling to the scalar curvature.

$$\widetilde{\mathcal{L}}_0 = \sqrt{-g}\mathcal{L}_0 \tag{2.1}$$

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - m^2\phi^\dagger\phi - \xi R\phi^\dagger\phi - \frac{\lambda}{2}(\phi^\dagger\phi)^2 \tag{2.2}$$

where

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.3}$$

$$D_\mu\phi = \partial_\mu\phi - ieA_\mu\phi \tag{2.4}$$

In this paper we study quantum effects of the gauge and scalar fields in the presence of background fields which are spacetime-dependent in general. We parametrize the complex scalar field in polar variables and shift the modulus field by the classical component ϕ_c,

$$\phi = \frac{1}{\sqrt{2}}(\phi_c + \rho)exp\left[i\frac{\pi}{\phi_c}\right] \tag{2.5}$$

To avoid the mixing between A_μ and π, we adopt the following gauge fixing and ghost part of the Lagrangian density:

$$\widetilde{\mathcal{L}}_G = \sqrt{-g}\mathcal{L}_G \tag{2.6}$$

$$\begin{aligned}
\mathcal{L}_G &= -i\boldsymbol{\delta}_B\left[-\left(\partial^\mu \frac{\bar{c}}{\phi_c^2}\right)\phi_c^2 A_\mu + \alpha\bar{c}\left(e\phi_c\pi + \frac{B}{2}\right)\right] && (2.7)\\
&= -\left(\partial^\mu \frac{B}{\phi_c^2}\right)\phi_c^2 A_\mu + \alpha e\phi_c B\pi + \frac{\alpha}{2}B^2 \\
&\quad +i\left\{-\phi_c^2\left(\partial_\mu \frac{\bar{c}}{\phi_c^2}\right)\partial^\mu c + \alpha e^2\phi_c^2\bar{c}c\right\} && (2.8)
\end{aligned}$$

where $\boldsymbol{\delta}_B$ represents the BRST transformation :

$$\boldsymbol{\delta}_B A_\mu = \partial_\mu c \tag{2.9}$$

$$\boldsymbol{\delta}_B \pi = e\phi_c c \tag{2.10}$$

$$\boldsymbol{\delta}_B \bar{c} = iB \tag{2.11}$$

$$\boldsymbol{\delta}_B(\text{otherwise}) = 0 \tag{2.12}$$

(when $g_{\mu\nu} = \eta_{\mu\nu}$ and $\phi_c =$ const., this gauge coincides with the R_ξ gauge [10][11].)

The total Lagrangian density is hence given by

$$\widetilde{\mathcal{L}} = \sqrt{-g}\,[\mathcal{L}_{\phi_c} + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_I] \tag{2.13}$$

$$\mathcal{L}_{\phi_c} = \frac{1}{2}(\partial_\mu\phi_c)^2 - \frac{1}{2}(m^2+\xi R)\phi_c^2 - \frac{\lambda}{8}\phi_c^4 \tag{2.14}$$

$$\mathcal{L}_1 = \partial_\mu\phi_c\partial^\mu\rho - \left\{(m^2+\xi R)\phi_c + \frac{\lambda}{2}\phi_c^3\right\}\rho \tag{2.15}$$

$$\begin{aligned}
\mathcal{L}_2 &= \frac{1}{2}(\partial_\mu\rho)^2 - \frac{1}{2}\left(m^2+\xi R+\frac{3}{2}\lambda\phi_c^2\right)\rho^2 \\
&\quad -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2\left(A_\mu - \partial_\mu\frac{\pi}{M}\right)^2 \\
&\quad -M^2\left(\partial^\mu\frac{B}{M^2}\right)A_\mu + \alpha MB\pi + \frac{\alpha}{2}B^2 \\
&\quad +i\left\{-M^2\left(\partial_\mu\frac{\bar{c}}{M^2}\right)\partial^\mu c + \alpha M^2\bar{c}c\right\} && (2.16)
\end{aligned}$$

$$
\begin{aligned}
\mathcal{L}_I &= -\frac{\lambda}{8}\rho^4 - \frac{\lambda}{2}\phi_c\rho^3 \\
&\quad +\frac{1}{2}(e^2\rho^2 + 2eM\rho)\left(A_\mu - \partial_\mu\frac{\pi}{M}\right)^2
\end{aligned}
\tag{2.17}
$$

where

$$
M = e\phi_c \tag{2.18}
$$

Note that M is spacetime-dependent in general.

Field equations are given by

$$
\Box A_\mu - \nabla^\nu\nabla_\mu A_\nu + (M+e\rho)^2\left(A_\mu - \partial_\mu\frac{\pi}{M}\right) - M^2\partial_\mu\frac{\breve{B}}{M} = 0 \tag{2.19}
$$

$$
\frac{\alpha}{M}(\breve{B}+\pi) + \frac{1}{M^4}\nabla^\mu(M^2A_\mu) + 0 \tag{2.20}
$$

$$
\frac{1}{M}\nabla^\mu\left\{(M+e\rho)^2\left(A_\mu - \partial_\mu\frac{\pi}{M}\right)\right\} + \alpha M^2\breve{B} = 0 \tag{2.21}
$$

$$
\frac{1}{M}\nabla^\mu\left\{M^2\left(\partial_\mu\frac{\breve{c}}{M}\right)\right\} + \alpha M^2\breve{c} = 0 \tag{2.22}
$$

$$
\frac{1}{M}\nabla^\mu\left\{M^2\left(\partial_\mu\frac{\breve{\bar{c}}}{M}\right)\right\} + \alpha M^2\breve{\bar{c}} = 0 \tag{2.23}
$$

where

$$
\breve{B} = \frac{B}{M} \quad , \qquad \breve{c} = Mc \quad , \qquad \breve{\bar{c}} = \frac{\bar{c}}{M} \tag{2.24}
$$

When the classical component ϕ_c is a solution of the following classical field equation:

$$
\Box\phi_c + (m^2+\xi R)\phi_c + \frac{\lambda}{2}\phi_c^3 = 0 \tag{2.25}
$$

$\mathcal{L}_1$ vanishes. The field equation of ρ is then given by

$$
\Box\rho + \left(m^2 + \xi R + \frac{3}{2}\lambda\phi_c^2\right)\rho + \frac{\lambda}{2}\rho^3 + \frac{3}{2}\lambda\phi_c\rho^2 - e(M+e\rho)\left(A_\mu - \partial_\mu\frac{\pi}{M}\right)^2 = 0 \tag{2.26}
$$

$\widetilde{\mathcal{L}}$ is invariant under the BRST transformation. The conserved Neother current is

$$j_{B\,\mu} = \frac{1}{\sqrt{-g}} \left\{ \frac{\partial \widetilde{\mathcal{L}}}{\partial \, \nabla^\mu A^\nu} \partial^\nu c + \frac{\partial \widetilde{\mathcal{L}}}{\partial \, \partial^\mu \pi} M c + i B (\partial / \partial \, \partial^\mu \bar{c}) \widetilde{\mathcal{L}} \right\} \tag{2.27}$$

from which the conserved charge is given as

$$\begin{aligned} \boldsymbol{Q}_B &= \int d^3x \, \sqrt{-g} \, j_B^{\,0} \\ &= \int d^3x \, \sqrt{-g} \, \{ \breve{B} \overset{\leftrightarrow}{\partial}{}^0 \breve{c} \} \end{aligned} \tag{2.28}$$

with the help of the field equations. $\widetilde{\mathcal{L}}$ is also invariant under the scale transformation:

$$c \quad \rightarrow \quad e^\theta c \tag{2.29}$$

$$\bar{c} \quad \rightarrow \quad e^{-\theta} \bar{c} \tag{2.30}$$

The conserved current and the charge of this transformation are given by

$$\begin{aligned} j_{c\,\mu} &= \frac{1}{\sqrt{-g}} \left\{ (\partial \widetilde{\mathcal{L}} / \partial \, \partial^\mu c) c + (-\bar{c}) (\partial / \partial \, \partial^\mu \bar{c}) \widetilde{\mathcal{L}} \right\} \\ &= i \{ \breve{\bar{c}} \overset{\leftrightarrow}{\partial}_\mu \breve{c} \} \end{aligned} \tag{2.31}$$

$$\begin{aligned} \boldsymbol{Q}_c &= \int d^3x \, \sqrt{-g} \, j_c^{\,0} \\ &= i \int d^3x \, \sqrt{-g} \, \{ \breve{\bar{c}} \overset{\leftrightarrow}{\partial}{}^0 \breve{c} \} \end{aligned} \tag{2.32}$$

where

$$\varphi \overset{\leftrightarrow}{\partial}_\mu \psi = \varphi \, \partial_\mu \psi - (\partial_\mu \varphi) \, \psi \tag{2.33}$$

Canonical momenta Π_ρ, Π^i, Π_B, Π_π, Π_c and $\Pi_{\bar{c}}$ conjugate to ρ, A_i, B, π, c and $\bar{c}$, respectively, are

$$\Pi_\rho = \sqrt{-g} \, \partial^0 \rho \tag{2.34}$$

$$\Pi^i = \sqrt{-g} \, (\partial^i A^0 - \partial^0 A^i) \tag{2.35}$$

$$\Pi_B = -\sqrt{-g}\ A^0 \tag{2.36}$$

$$\Pi_\pi = \frac{\sqrt{-g}}{M}\ (M + e\rho)^2 \left(\partial^0 \frac{\pi}{M} - A^0\right) \tag{2.37}$$

$$\Pi_c = -i\sqrt{-g}\ M^2 \partial^0 \frac{\bar{c}}{M^2} \tag{2.38}$$

$$\Pi_{\bar{c}} = i\sqrt{-g}\ \partial^0 c \tag{2.39}$$

We set up Canonical (anti-)commutation relations as

$$[\Pi_\rho(x)\ ,\ \rho(y)]_{\mathrm{E.T.}} = -i\delta^3(x - y) \tag{2.40}$$

$$[\Pi^i(x)\ ,\ A_j(y)]_{\mathrm{E.T.}} = -i\delta^i_{\ j}\delta^3(x - y) \tag{2.41}$$

$$[\Pi_B(x)\ ,\ B(y)]_{\mathrm{E.T.}} = -i\delta^3(x - y) \tag{2.42}$$

$$[\Pi_\pi(x)\ ,\ \pi(y)]_{\mathrm{E.T.}} = -i\delta^3(x - y) \tag{2.43}$$

$$\{\Pi_c(x)\ ,\ c(y)\}_{\mathrm{E.T.}} = -i\delta^3(x - y) \tag{2.44}$$

$$\{\Pi_{\bar{c}}(x)\ ,\ \bar{c}(y)\}_{\mathrm{E.T.}} = -i\delta^3(x - y) \tag{2.45}$$

III Extraction of a massive vector field

The Laglangian density $\widetilde{\mathcal{L}}$ should be diagonalized. A physical massive vector field, which is denoted as U_μ, is a linear combination of A_μ, π and B.

$$U_\mu = A_\mu - \partial_\mu \frac{\pi}{M} - \partial_\mu \frac{\breve{B}}{M} \tag{3.1}$$

From the BRST transformation of A_μ, π and B, U_μ is shown to be BRST-invariant. In terms of U_μ, the Laglangian density $\widetilde{\mathcal{L}}$ can be rewritten as

$$\widetilde{\mathcal{L}} = \sqrt{-g}\ [\mathcal{L}_{\phi_c} + \mathcal{L}_1 + \mathcal{L}_2' + \mathcal{L}_I'] \tag{3.2}$$

$$
\begin{aligned}
\mathcal{L}_2' &= \frac{1}{2}(\partial_\mu \rho)^2 - \frac{1}{2}\left(m^2 + \xi R + \frac{3}{2}\lambda\phi_c^2\right)\rho^2 \\
&\quad -\frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{1}{2}M^2 U_\mu U^\mu \\
&\quad +\frac{1}{2}\left\{-M^2\left(\partial_\mu \frac{\breve{B}}{M}\right)\left(\partial^\mu \frac{\breve{B}}{M}\right) + \alpha M^2 \breve{B}^2\right\} \\
&\quad +\left\{-M^2\left(\partial_\mu \frac{\breve{B}}{M}\right)\left(\partial^\mu \frac{\pi}{M}\right) + \alpha M^2 \breve{B}\pi\right\} \\
&\quad +i\left\{-M^2\left(\partial_\mu \frac{\breve{\bar{c}}}{M}\right)\left(\partial^\mu \frac{\breve{c}}{M}\right) + \alpha M^2 \breve{\bar{c}}\breve{c}\right\}
\end{aligned}
\tag{3.3}
$$

$$
\mathcal{L}_I' = -\frac{\lambda}{8}\rho^4 - \frac{\lambda}{2}\phi_c\rho^3 + \frac{1}{2}(e^2\rho^2 + 2eM\rho)\left(U_\mu + \partial_\mu \frac{\breve{B}}{M}\right)^2 \tag{3.4}
$$

where

$$
H_{\mu\nu} = \nabla_\mu U_\nu - \nabla_\nu U_\mu = \partial_\mu U_\nu - \partial_\nu U_\mu = F_{\mu\nu} \tag{3.5}
$$

In the rest of this paper we shall be mainly concerned with effects of spacetime dependence in the background scalar field ϕ_c and the background metric $g_{\mu\nu}$ to these free fields. The interaction part of the Lagrangian density can be handled as a perturbation but shall not be studied in detail. Free field equations derived from the bilinear part of the Laglangian density are hence given by

$$
\Box U_\mu - \nabla^\nu \nabla_\mu U_\nu + M^2 U_\mu = 0 \tag{3.6}
$$

$$
\Box \breve{B} + \left(\alpha M^2 - \frac{\Box M}{M}\right)\breve{B} = 0 \tag{3.7}
$$

$$
\Box \pi + \left(\alpha M^2 - \frac{\Box M}{M}\right)\pi = 0 \tag{3.8}
$$

$$
\Box \breve{c} + \left(\alpha M^2 - \frac{\Box M}{M}\right)\breve{c} = 0 \tag{3.9}
$$

$$
\Box \breve{\bar{c}} + \left(\alpha M^2 - \frac{\Box M}{M}\right)\breve{\bar{c}} = 0 \tag{3.10}
$$

When ϕ_c is a solution of the classical field equation, the field equation of ρ becomes

$$
\Box \rho + \left(m^2 + \xi R + \frac{3}{2}\lambda\phi_c^2\right)\rho = 0 \tag{3.11}
$$

At this level, each free field corresponds to a one particle state. Therefore we call ρ and U_μ physical fields in strong sense and call B, π, c and $\bar{c}$ a BRST quartet.

IV Basis functions and innerproducts

We expand the quantum fields in normal modes.

$$U_\mu(x) = \sum_{k,\,a} \Big\{ \boldsymbol{U}(ka)\, f_\mu(ka|x) + \boldsymbol{U}^\dagger(ka)\, f^*_\mu(ka|x) \Big\} \tag{4.1}$$

$$B(x) = M(x) \sum_k \Big\{ \boldsymbol{B}(k)\, g_B(k|x) + \boldsymbol{B}^\dagger(k)\, g^*_B(k|x) \Big\} \tag{4.2}$$

$$\pi(x) = \sum_k \Big\{ \boldsymbol{\pi}(k)\, g_\pi(k|x) + \boldsymbol{\pi}^\dagger(k)\, g^*_\pi(k|x) \Big\} \tag{4.3}$$

$$c(x) = \frac{1}{M(x)} \sum_k \Big\{ \boldsymbol{c}(k)\, g_c(k|x) + \boldsymbol{c}^\dagger(k)\, g^*_c(k|x) \Big\} \tag{4.4}$$

$$\bar{c}(x) = M(x) \sum_k \Big\{ \bar{\boldsymbol{c}}(k)\, g_{\bar{c}}(k|x) + \bar{\boldsymbol{c}}^\dagger(k)\, g^*_{\bar{c}}(k|x) \Big\} \tag{4.5}$$

$$\rho(x) = \sum_k \Big\{ \boldsymbol{\rho}(k)\, h(k|x) + \boldsymbol{\rho}^\dagger(k)\, h^*(k|x) \Big\} \tag{4.6}$$

Here k and a are a set of quantum numbers to label modes. In Minkowski space, they are related to momentum and spin . $(a = 1,\ 2,\ 3)$

Basis functions satisfy the following wave equations:

$$\begin{cases} \Box f_\mu - \nabla^\nu \nabla_\mu f_\nu + M^2 f_\mu = 0 \\ \nabla^\mu (M^2 f_\mu) = 0 \end{cases} \tag{4.7}$$

$$\Box g_F + \left(\alpha M^2 - \frac{\Box M}{M} \right) g_F = 0 \tag{4.8}$$

$$\Box h + \left(m^2 + \xi R + \frac{3}{2} \lambda \phi_c \right) h = 0 \tag{4.9}$$

where $F = B,\ \pi,\ c,\ \bar{c}$

Let us introduce innerproducts for basis functions. We can show that

$$\begin{aligned} \nabla_\nu [f^*_\mu(ka|x) \{ \nabla^\nu f^\mu(k'a'|x) \ &- \ \nabla^\mu f^\nu(k'a'|x) \} \\ &- \ \{ \nabla^\nu f^{\mu *}(ka|x) - \nabla^\mu f^{\nu *}(ka|x) \} f_\mu(k'a'|x)] = 0 \end{aligned} \tag{4.10}$$

$$\nabla_\nu [g_B^*(k|x) \overset{\leftrightarrow}{\partial}{}^\nu g_\pi(k'|x)] = 0 \tag{4.11}$$

$$\nabla_\nu [g_{\bar{c}}^*(k|x) \overset{\leftrightarrow}{\partial}{}^\nu g_c(k'|x)] = 0 \tag{4.12}$$

$$\nabla_\nu [h^*(k|x) \overset{\leftrightarrow}{\partial}{}^\nu h(k'|x)] = 0 \tag{4.13}$$

by using the wave equations. Thus we can define the time-independent innerproducts as

$$\begin{aligned} \langle\langle f(ka),\ f(k'a')\rangle\rangle \ &= \ -i\int d^3x\,\sqrt{-g}\,[f_\mu^*(ka|x)\{\nabla^0 f^\mu(k'a'|x) - \nabla^\mu f^0(k'a'|x)\} \\ &\qquad - \{\nabla^0 f^{\mu *}(ka|x) - \nabla^\mu f^{0*}(ka|x)\} f_\mu(k'a'|x)] \end{aligned} \tag{4.14}$$

$$\langle\langle g_B(k),\ g_\pi(k')\rangle\rangle = i\int d^3x\,\sqrt{-g}\,[g_B^*(k|x) \overset{\leftrightarrow}{\partial}{}^0 g_\pi(k'|x)] \tag{4.15}$$

$$\langle\langle g_{\bar{c}}(k),\ g_c(k')\rangle\rangle = i\int d^3x\,\sqrt{-g}\,[g_{\bar{c}}^*(k|x) \overset{\leftrightarrow}{\partial}{}^0 g_c(k'|x)] \tag{4.16}$$

$$\langle\langle h(k),\ h(k')\rangle\rangle = i\int d^3x\,\sqrt{-g}\,[h^*(k|x) \overset{\leftrightarrow}{\partial}{}^0 h(k'|x)] \tag{4.17}$$

We impose the following orthonormality:

$$\langle\langle f(ka),\ f(k'a')\rangle\rangle = \delta(k,\ k')\delta(a,\ a'), \quad \langle\langle f^*(ka),\ f(k'a')\rangle\rangle = 0 \tag{4.18}$$

$$\langle\langle g_B(k),\ g_\pi(k')\rangle\rangle = \delta(k,\ k'), \quad \langle\langle g_B^*(k),\ g_\pi(k')\rangle\rangle = 0 \tag{4.19}$$

$$\langle\langle g_{\bar{c}}(k),\ g_c(k')\rangle\rangle = \delta(k,\ k'), \quad \langle\langle g_{\bar{c}}^*(k),\ g_c(k')\rangle\rangle = 0 \tag{4.20}$$

$$\langle\langle h(k),\ h(k')\rangle\rangle = \delta(k,\ k'), \quad \langle\langle h^*(k),\ h(k')\rangle\rangle = 0 \tag{4.21}$$

By using the innerproducts, we can express the coefficients as

$$\boldsymbol{U}(ka) = \langle\langle f(ka),\ U\rangle\rangle\,, \quad \boldsymbol{U}^\dagger(ka) = -\,\langle\langle f^*(ka),\ U\rangle\rangle \tag{4.22}$$

$$\boldsymbol{B}(k) = \langle\langle g_\pi(k),\ B\rangle\rangle\,, \quad \boldsymbol{B}^\dagger(k) = -\,\langle\langle g_\pi^*(k),\ B\rangle\rangle \tag{4.23}$$

$$\boldsymbol{\pi}(k) = \langle\langle g_B(k),\ \pi\rangle\rangle\,, \quad \boldsymbol{\pi}^\dagger(k) = -\,\langle\langle g_B^*(k),\ \pi\rangle\rangle \tag{4.24}$$

$$\boldsymbol{c}(k) = \langle\langle g_{\bar{c}}(k),\ c\rangle\rangle\,, \quad \boldsymbol{c}^\dagger(k) = -\,\langle\langle g_{\bar{c}}^*(k),\ c\rangle\rangle \tag{4.25}$$

$$\bar{\boldsymbol{c}}(k) = \langle\langle g_c(k),\ \bar{c}\rangle\rangle\,, \quad \bar{\boldsymbol{c}}^\dagger(k) = -\,\langle\langle g_c^*(k),\ \bar{c}\rangle\rangle \tag{4.26}$$

$$\boldsymbol{\rho}(k) = \langle\langle h(k),\ \rho\rangle\rangle\,, \quad \boldsymbol{\rho}^\dagger(k) = -\,\langle\langle h^*(k),\ \rho\rangle\rangle \tag{4.27}$$

From the canonical (anti-)commutation relations, we obtain (anti-)commutation relations among coefficients.

$$[\boldsymbol{U}(ka),\ \boldsymbol{U}^\dagger(k'a')] = \delta(k,\ k')\delta(a,\ a') \tag{4.28}$$

$$[\boldsymbol{B}(k),\ \boldsymbol{\pi}^\dagger(k')] = [\boldsymbol{\pi}(k),\ \boldsymbol{B}^\dagger] = -\delta(k,\ k') \tag{4.29}$$

$$[\boldsymbol{\pi}(k),\ \boldsymbol{\pi}^\dagger(k')] = \delta(k,\ k') \tag{4.30}$$

$$\{\boldsymbol{c}(k),\ \bar{\boldsymbol{c}}^\dagger(k')\} = -\{\boldsymbol{c}^\dagger(k),\ \bar{\boldsymbol{c}}(k')\} = i\delta(k,\ k') \tag{4.31}$$

$$[\boldsymbol{\rho}(k),\ \boldsymbol{\rho}^\dagger(k')] = \delta(k,\ k') \tag{4.32}$$

$$\text{otherwise} = 0 \tag{4.33}$$

V Vacuum states

In this section, we shall construct the "in Fock space" $\mathcal{V}_{in}$ and the "out Fock space" $\mathcal{V}_{out}$, and investigate the relation between them.

If basis functions f, g_F and h are positive frequency solutions in the region $t \to -\infty$, we write them as f_{in}, $g_{F\,in}$ and h_{in}. In a parallel way, we can define another basis, f_{out}, $g_{F\,out}$ and h_{out} which are positive frequency solutions in the region $t \to \infty$. When there is a unique positive frequency solution, we have the following relation:

$$g_{in} \equiv g_{B\,in} = g_{\pi\,in} = g_{c\,in} = g_{\bar{c}\,in} \tag{5.1}$$

because they satisfy the same equation and the same boundary condition. Similarly, we also have

$$g_{out} \equiv g_{B\,out} = g_{\pi\,out} = g_{c\,out} = g_{\bar{c}\,out} \tag{5.2}$$

But, when the positive frequency solutions are not unique, it is not obvious what condition should be imposed. We shall pick out this from the BRST-invariance of vacuum states.

As is well known, the basis $\{f_{in},\, g_{F\,in},\, h_{in}\,;\, f^*_{in},\, g_{F}{}^*_{in},\, h^*_{in}\}$ are in general different from the basis $\{f_{out},\, g_{F\,out},\, h_{out}\,;\, f^*_{out},\, g_{F}{}^*_{out},\, h^*_{out}\}$ in the presence of spacetime-dependent background fields. We can expand the fields in two ways. Using the orthonormality of basis functions, the coefficients of $\{f_{in},\, g_{F\,in},\, h_{in}\,;\, f^*_{in},\, g_{F}{}^*_{in},\, h^*_{in}\}$ are related with the coefficients of $\{f_{out},\, g_{F\,out},\, h_{out}\,;\, f^*_{out},\, g_{F}{}^*_{out},\, h^*_{out}\}$ by the following Bogoliubov transformation:

$$\boldsymbol{U}_{in}(ka) = \sum_{k',\,a'} \Big\{ {\alpha_U}^*(ka, k'a')\, \boldsymbol{U}_{out}(k'a') - {\beta_U}^*(ka, k'a')\, \boldsymbol{U}^\dagger{}_{out}(k'a') \Big\} \tag{5.3}$$

$$\boldsymbol{B}_{in}(k) = \sum_{k'} \Big\{ \bar{\alpha}^*_{B\,\pi}(k, k')\, \boldsymbol{B}_{out}(k') - \bar{\beta}^*_{B\,\pi}(k, k')\, \boldsymbol{B}^\dagger{}_{out}(k') \Big\} \tag{5.4}$$

$$\boldsymbol{\pi}_{in}(k) = \sum_{k'} \Big\{ {\alpha_{B\,\pi}}^*(k, k')\, \boldsymbol{\pi}_{out}(k') - {\beta_{B\,\pi}}^*(k, k')\, \boldsymbol{\pi}^\dagger{}_{out}(k') \Big\} \tag{5.5}$$

$$\boldsymbol{c}_{in}(k) = \sum_{k'} \Big\{ {\alpha_{\bar{c}\,c}}^*(k, k')\, \boldsymbol{c}_{out}(k') - {\beta_{\bar{c}\,c}}^*(k, k')\, \boldsymbol{c}^\dagger{}_{out}(k') \Big\} \tag{5.6}$$

$$\bar{\boldsymbol{c}}_{in}(k) = \sum_{k'} \Big\{ \bar{\alpha}^*_{\bar{c}\,c}(k, k')\, \bar{\boldsymbol{c}}_{out}(k') - \bar{\beta}^*_{\bar{c}\,c}(k, k')\, \bar{\boldsymbol{c}}^\dagger_{out}(k') \Big\} \tag{5.7}$$

$$\boldsymbol{\rho}_{in}(k) = \sum_{k'} \Big\{ {\alpha_\rho}^*(k, k')\, \boldsymbol{\rho}_{out}(k') - {\beta_\rho}^*(k, k')\, \boldsymbol{\rho}^\dagger{}_{out}(k') \Big\} \tag{5.8}$$

where

$$\alpha_U(ka, k'a') = \langle\langle f_{out}(k'a'),\ f_{in}(ka) \rangle\rangle \tag{5.9}$$

$$\beta_U(ka, k'a') = -\,\langle\langle f^*_{out}(k'a'),\ f_{in}(ka) \rangle\rangle \tag{5.10}$$

$$\alpha_{B\,\pi}(k, k') = \langle\langle g_{\pi\,out}(k'),\ g_{B\,in}(k) \rangle\rangle \tag{5.11}$$

$$\beta_{B\,\pi}(k, k') = -\,\langle\langle g^*_{\pi\,out}(k'),\ g_{B\,in}(k) \rangle\rangle \tag{5.12}$$

$$\bar{\alpha}_{B\,\pi}(k, k') = \langle\langle g_{B\,out}(k'),\ g_{\pi\,in}(k) \rangle\rangle \tag{5.13}$$

$$\bar{\beta}_{B\,\pi}(k, k') = -\,\langle\langle g^*_{B\,out}(k'),\ g_{\pi\,in}(k) \rangle\rangle \tag{5.14}$$

$$\alpha_{\bar{c}\,c}(k, k') = \langle\langle g_{c\,out}(k'),\ g_{\bar{c}\,in}(k) \rangle\rangle \tag{5.15}$$

$$\beta_{\bar{c}\,c}(k, k') = -\,\langle\langle g^*_{c\,out}(k'),\ g_{\bar{c}\,in}(k) \rangle\rangle \tag{5.16}$$

$$\bar{\alpha}_{\bar{c}\,c}(k, k') = \langle\langle g_{\bar{c}\,out}(k'),\ g_{c\,in}(k) \rangle\rangle \tag{5.17}$$

$$\bar{\beta}_{\bar{c}c}(k,k') = -\langle\langle g^{*}_{\bar{c}\,out}(k'),\; g_{c\,in}(k)\rangle\rangle \tag{5.18}$$

$$\alpha_{\rho}(k,k') = \langle\langle h_{out}(k'),\; h_{in}(k)\rangle\rangle \tag{5.19}$$

$$\beta_{\rho}(k,k') = -\langle\langle h^{*}_{out}(k'),\; h_{in}(k)\rangle\rangle \tag{5.20}$$

Now we can construct the "in Fock space" $\mathcal{V}_{in}$ and the "out Fock space" $\mathcal{V}_{out}$. The in-vacuum is characterized by

$$\boldsymbol{U}_{in}|0_{in}\rangle = \boldsymbol{B}_{in}|0_{in}\rangle = \boldsymbol{\pi}_{in}|0_{in}\rangle = \boldsymbol{c}_{in}|0_{in}\rangle = \bar{\boldsymbol{c}}_{in}|0_{in}\rangle = \boldsymbol{\rho}_{in}|0_{in}\rangle = 0 \tag{5.21}$$

The in Fock space is obtained by applying in-creation operators to $|0_{in}\rangle$.

$$\mathcal{V}_{in} = \Big\{\cdots \boldsymbol{U}^{\dagger}{}_{in}\cdots \boldsymbol{B}^{\dagger}{}_{in}\cdots \boldsymbol{\pi}^{\dagger}{}_{in}\cdots \boldsymbol{c}^{\dagger}{}_{in}\cdots \bar{\boldsymbol{c}}^{\dagger}_{in}\cdots \boldsymbol{\rho}^{\dagger}{}_{in}\cdots |0_{in}\rangle\Big\} \tag{5.22}$$

Similarly the out-vacuum is defined by

$$\boldsymbol{U}_{out}|0_{out}\rangle = \boldsymbol{B}_{out}|0_{out}\rangle = \boldsymbol{\pi}_{out}|0_{out}\rangle = \boldsymbol{c}_{out}|0_{out}\rangle = \bar{\boldsymbol{c}}_{out}|0_{out}\rangle = \boldsymbol{\rho}_{out}|0_{out}\rangle = 0 \tag{5.23}$$

and the out Fock space is

$$\mathcal{V}_{out} = \Big\{\cdots \boldsymbol{U}^{\dagger}{}_{out}\cdots \boldsymbol{B}^{\dagger}{}_{out}\cdots \boldsymbol{\pi}^{\dagger}{}_{out}\cdots \boldsymbol{c}^{\dagger}{}_{out}\cdots \bar{\boldsymbol{c}}^{\dagger}_{out}\cdots \boldsymbol{\rho}^{\dagger}{}_{out}\cdots |0_{out}\rangle\Big\} \tag{5.24}$$

In general, $|0_{in}\rangle \neq |0_{out}\rangle$. Because the coefficients of $\{f_{in},\; g_{F\,in},\; h_{in}\;;\; f^{*}_{in},\; g_{F}{}^{*}_{in},\; h^{*}_{in}\}$ are related with the coefficients of $\{f_{out},\; g_{F\,out},\; h_{out}\;;\; f^{*}_{out},\; g_{F}{}^{*}_{out},\; h^{*}_{out}\}$ by the Bogoliubov transformation, the in-vacuum can be expressed in terms of the out-vacuum.

From the definition of the in-vacuum,

$$
\begin{aligned}
|0_{in}\rangle \propto \quad & exp\Bigg[\sum_{\{k,\,a\}\,\{k',\,a'\}} \frac{1}{2}\lambda^*_U(ka,k'a')\boldsymbol{U}^\dagger{}_{out}(ka)\boldsymbol{U}^\dagger{}_{out}(k'a') \\
& - \sum_{k,\,k'} \lambda^*_{B\,\pi}(k,\ k')\left\{\frac{1}{2}\boldsymbol{B}^\dagger{}_{out}(k)\boldsymbol{B}^\dagger{}_{out}(k') + \boldsymbol{B}^\dagger{}_{out}(k)\boldsymbol{\pi}^\dagger{}_{out}(k')\right\} \\
& - \sum_{k,\,k'} i\lambda^*_{\bar{c}\,c}(k,\ k')\bar{\boldsymbol{c}}^\dagger_{out}(k)\boldsymbol{c}^\dagger{}_{out}(k') \\
& + \sum_{k,\,k'} \frac{1}{2}\lambda^*_\rho(k,\ k')\boldsymbol{\rho}^\dagger{}_{out}(k)\boldsymbol{\rho}^\dagger{}_{out}(k')\Bigg]\,|0_{out}\rangle
\end{aligned} \tag{5.25}
$$

where

$$\lambda_U(ka,k'a') = (\alpha_U{}^{-1}\beta_U)(ka,k'a') \tag{5.26}$$

$$\lambda_{B\,\pi}(k,k') = (\alpha_{B\,\pi}{}^{-1}\beta_{B\,\pi})(k,k') = (\bar{\alpha}^{-1}_{B\,\pi}\bar{\beta}_{B\,\pi})(k',k) = \bar{\lambda}_{B\,\pi}(k',k) \tag{5.27}$$

$$\lambda_{\bar{c}\,c}(k,k') = (\alpha_{\bar{c}\,c}{}^{-1}\beta_{\bar{c}\,c})(k,k') = (\bar{\alpha}^{-1}_{\bar{c}\,c}\bar{\beta}_{\bar{c}\,c})(k',k) = \bar{\lambda}_{\bar{c}\,c}(k',k) \tag{5.28}$$

$$\lambda_\rho(k,k') = (\alpha_\rho{}^{-1}\beta_\rho)(k,k') \tag{5.29}$$

(From the unitarity condition of the Bogoliubov coefficients, λ_U, $\lambda_{B\,\pi}$, $\bar{\lambda}_{B\,\pi}$ and λ_ρ are symmetric. i.e., $\lambda(k,k') = \lambda(k',k)$. The proof is given in Appendix.) This formula is the generalization of that given by [12] in which RW metric is assumed and the (anti-)commutation relations of field operators are diagonal.

From this formula, the in-vacuum can be regarded as the state in which out-particles including unphysical particles condense on the out-vacuum. One might worry about that the in-vacuum could be a unphysical state in the system described by the bilinear part of the Laglangian density. In the rest of this section, we shall show what condition should be chosen for BRST-invariant vacuum states.

Now we follow the formalism given in [13], in which the physical states are selected by the subsidiary condition:

$$\boldsymbol{Q}_B|phys.\rangle = 0 \tag{5.30}$$

The BRST charge in terms of annihilation and creation operators is

$$
\begin{aligned}
\boldsymbol{Q}_{Bin} = (-i) \sum_{k,\,k'} & \Big\{ \boldsymbol{B}_{in(out)}(k) \Big\langle\Big\langle g^*_{B\,in(out)}(k),\ g^*_{c\,in(out)}(k') \Big\rangle\Big\rangle\, \boldsymbol{c}^\dagger{}_{in(out)}(k') \\
+ & \ \boldsymbol{B}^\dagger{}_{in(out)}(k) \Big\langle\Big\langle g_{B\,in(out)}(k),\ g_{c\,in(out)}(k') \Big\rangle\Big\rangle\, \boldsymbol{c}_{in(out)}(k') \\
+ & \ \boldsymbol{B}_{in(out)}(k) \Big\langle\Big\langle g^*_{B\,in(out)}(k),\ g_{c\,in(out)}(k') \Big\rangle\Big\rangle\, \boldsymbol{c}_{in(out)}(k') \\
+ & \ \boldsymbol{B}^\dagger{}_{in(out)}(k) \Big\langle\Big\langle g_{B\,in(out)}(k),\ g^*_{c\,in(out)}(k') \Big\rangle\Big\rangle\, \boldsymbol{c}^\dagger{}_{in(out)}(k') \Big\}
\end{aligned} \tag{5.31}
$$

So that the in(out)-vacuum is a physical state, the following condition is necessary and sufficient.

$$
\exists z_{in(out)}(k,k') \quad : \quad det\ z_{in(out)} \neq 0
$$
$$
\begin{cases}
g_{B\,in(out)}(k) = \sum_{k'} z^*_{in(out)}(k,k') g_{\bar{c}\,in(out)}(k') \\
g_{\pi\,in(out)}(k) = \sum_{k'} z^{-1}_{in(out)}(k',k) g_{c\,in(out)}(k')
\end{cases} \tag{5.32}
$$

If both vacuum states are physical, we have the following relation:

$$
\boldsymbol{Q}_B = \boldsymbol{Q}_{Bin} = \boldsymbol{Q}_{Bout} \tag{5.33}
$$

where

$$
\boldsymbol{Q}_{Bin} = i \sum_{k,\,k'} \Big\{ \boldsymbol{B}_{in}(k) z^*_{in}(k,k') \boldsymbol{c}^\dagger{}_{in}(k') - \boldsymbol{B}^\dagger{}_{in}(k) z_{in}(k,k') \boldsymbol{c}_{in}(k') \Big\} \tag{5.34}
$$

$$
\boldsymbol{Q}_{Bout} = i \sum_{k,\,k'} \Big\{ \boldsymbol{B}_{out}(k) z^*_{out}(k,k') \boldsymbol{c}^\dagger{}_{out}(k') - \boldsymbol{B}^\dagger{}_{out}(k) z_{out}(k,k') \boldsymbol{c}_{out}(k') \Big\} \tag{5.35}
$$

In case $z_{in} = z_{out} = I$, the above formula was proved in [14] using the unitarity condition of the Bogoliubov coefficients. The authors of [14] concluded that the invariance of the BRST charge under the Bogoliubov transformation implies an absence of unphysical particles. However, we would rather claim that unphysical particles are condensed but do not contribute to physical processes such as backreaction to background fields, which we will prove in the next section.

Before completing this section, we shall again look the form of condensation when both vacuum states are physical. From (5.25) and (5.32),

$$\begin{aligned}|0_{in}\rangle \ \ &\propto \ \ exp\Bigg[\sum_{\{k,\,a\}\ \{k',\,a'\}} \frac{1}{2}\lambda^*_U(ka,k'a')\boldsymbol{U}^\dagger{}_{out}(ka)\boldsymbol{U}^\dagger{}_{out}(k'a') \\ &\qquad + \sum_{k,\,k'} \lambda^*_{BRST}(k,\ k')\left\{i\boldsymbol{Q}_{B\,out},\ i\bar{\boldsymbol{c}}^\dagger_{out}(k)\left(\boldsymbol{\pi}^\dagger{}_{out}(k') + \frac{1}{2}\boldsymbol{B}^\dagger{}_{out}(k')\right)\right\} \\ &\qquad + \sum_{k,\,k'} \frac{1}{2}\lambda^*_\rho(k,\ k')\boldsymbol{\rho}^\dagger{}_{out}(k)\boldsymbol{\rho}^\dagger{}_{out}(k')\Bigg]|0_{out}\rangle \end{aligned} \tag{5.36}$$

where

$$\lambda_{BRST}(k,k') = \sum_{k''} z^{*\,-1}_{out}(k,k'')\lambda_{B\pi}(k'',k') = \sum_{k''} \lambda_{\bar{c}c}(k,k'')z^{-1}_{out}(k'',k') \tag{5.37}$$

Consequently, in case both vacuum states are BRST-invariant, the condensation of the BRST quartet sector occur in a BRST-exact form and we must choose basis functions as

$$\begin{cases} g_{B\,in} = z^*_{in} g_{\bar{c}\,in}, \quad z^T_{in} g_{\pi\,in} = g_{c\,in} \\ g_{B\,out} = z^*_{in} g_{\bar{c}\,out}, \quad z^T_{out} g_{\pi\,out} = g_{c\,out} \end{cases} \tag{5.38}$$

where we use matrix representation.

VI Backreaction

Now we shall prove the fact that produced unphysical particles do not contribute to backreaction. We look at backreaction in the Einstein equations first. And next, backreaction in the effective equation of the classical component ϕ_c will be considered.

A The Einstein equations

The energy-momentum tensor derived from the bilinear part of the Laglangian density is given by

$$T_{2\,\mu\nu} = T_{U_\mu\,\mu\nu} + T_{\rho\,\mu\nu} + T_{B\pi\,\mu\nu} + T_{\bar{c}c\,\mu\nu} \tag{6.1}$$

where

$$T_{U_\mu\,\mu\nu} = -H_{\mu\sigma}H_\nu{}^\sigma + M^2U_\mu U_\nu + \frac{1}{4}g_{\mu\nu}\left(H_{\gamma\sigma}H^{\gamma\sigma} - 2M^2U_\sigma U^\sigma\right) \tag{6.2}$$

$$\begin{aligned} T_{\rho\,\mu\nu} &= \partial_\mu\rho\partial_\nu\rho + \frac{1}{2}g_{\mu\nu}\left\{(m^2+\xi R+\frac{3}{2}\lambda\phi_c^2)\rho^2 - \partial_\sigma\rho\partial^\sigma\rho\right\} \\ &\quad +2\xi\left\{g_{\mu\nu}(\rho\Box\rho+\partial_\sigma\rho\partial^\sigma\rho) - \frac{1}{2}\rho(\nabla_\mu\partial_\nu+\nabla_\nu\partial_\mu)\rho - \partial_\mu\rho\partial_\nu\rho - \frac{1}{2}R_{\mu\nu}\rho^2\right\} \end{aligned} \tag{6.3}$$

$$\begin{aligned} T_{B\pi\,\mu\nu} &= -M^2\left\{\partial_\mu\frac{\breve{B}}{M}\partial_\nu\frac{\breve{B}}{M} + \frac{1}{2}g_{\mu\nu}\left(\alpha\breve{B}^2 - \partial_\sigma\frac{\breve{B}}{M}\partial^\sigma\frac{\breve{B}}{M}\right)\right\} \\ &\quad -M^2\left\{\partial_\mu\frac{\breve{B}}{M}\partial_\nu\frac{\pi}{M} + \partial_\nu\frac{\breve{B}}{M}\partial_\mu\frac{\pi}{M} + g_{\mu\nu}\left(\alpha\breve{B}\pi - \partial_\sigma\frac{\breve{B}}{M}\partial^\sigma\frac{\pi}{M}\right)\right\} \end{aligned} \tag{6.4}$$

$$T_{\bar{c}c\,\mu\nu} = -iM^2\left\{\partial_\mu\frac{\breve{\bar{c}}}{M}\partial_\nu\frac{\breve{c}}{M} + \partial_\nu\frac{\breve{\bar{c}}}{M}\partial_\mu\frac{\breve{c}}{M} + g_{\mu\nu}\left(\alpha\breve{\bar{c}}\breve{c} - \partial_\sigma\frac{\breve{\bar{c}}}{M}\partial^\sigma\frac{\breve{c}}{M}\right)\right\} \tag{6.5}$$

The Einstein equations including 1-loop backreaction are

$$G_{\mu\nu} = -\kappa\left\{T_{\phi_c\,\mu\nu} + \langle T_{2\,\mu\nu}\rangle\right\} \tag{6.6}$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \tag{6.7}$$

$$\begin{aligned} T_{\phi_c\,\mu\nu} &= \partial_\mu\phi_c\partial_\nu\phi_c + \frac{1}{2}g_{\mu\nu}\left\{(m^2+\xi R)\phi_c^2 + \frac{\lambda}{4}\phi_c^4 - \partial_\sigma\phi_c\partial^\sigma\phi_c\right\} \\ &\quad +2\xi\left\{g_{\mu\nu}(\phi_c\Box\phi_c+\partial_\sigma\phi_c\partial^\sigma\phi_c) - \frac{1}{2}\phi_c(\nabla_\mu\partial_\nu+\nabla_\nu\partial_\mu)\phi_c\right. \\ &\qquad \left. -\partial_\mu\phi_c\partial_\nu\phi_c - \frac{1}{2}R_{\mu\nu}\phi_c^2\right\} \end{aligned} \tag{6.8}$$

In general, the contributions from $\langle T_{B\pi\,\mu\nu}\rangle$ and $\langle T_{\bar{c}c\,\mu\nu}\rangle$ don't vanish. But if the state is physical, they cancel each other because $T_{B\pi\,\mu\nu} + T_{\bar{c}c\,\mu\nu}$ is a BRST-exact operator.

$$T_{BRST\,\mu\nu} \equiv T_{B\pi\,\mu\nu} + T_{\bar{c}c\,\mu\nu} = \{i\boldsymbol{Q}_B,\ \Upsilon_{BRST\,\mu\nu}\} \tag{6.9}$$

where

$$\begin{aligned} \Upsilon_{BRST\,\mu\nu} &= iM^2\left[\partial_\mu\frac{\breve{\bar{c}}}{M}\partial_\nu\left(\frac{\pi}{M}+\frac{1}{2}\frac{\breve{B}}{M}\right) + \partial_\nu\frac{\breve{\bar{c}}}{M}\partial_\mu\left(\frac{\pi}{M}+\frac{1}{2}\frac{\breve{B}}{M}\right)\right. \\ &\quad \left. +g_{\mu\nu}\left\{\alpha\breve{\bar{c}}\left(\pi+\frac{1}{2}\breve{B}\right) - \partial_\sigma\frac{\breve{\bar{c}}}{M}\partial^\sigma\left(\frac{\pi}{M}+\frac{1}{2}\frac{\breve{B}}{M}\right)\right\}\right] \end{aligned} \tag{6.10}$$

Consequently, there is no backreaction from the produced BRST quartet.

Especially when the in-vacuum is physical, the cancellation can be explicitly shown as follows,

$$\begin{aligned}
\langle 0_{in}|T_{BRST\,\mu\nu}|0_{in}\rangle \;=\;& \sum_k M^2 \Bigg[\left(\partial_\mu \frac{g_{B\,in}(k)}{M}\right)\left(\partial_\nu \frac{g^*_{\pi\,in}(k)}{M}\right) + \left(\partial_\nu \frac{g_{B\,in}(k)}{M}\right)\left(\partial_\mu \frac{g^*_{\pi\,in}(k)}{M}\right) \\
&\quad + g_{\mu\nu}\left\{ \alpha\, g_{B\,in}(k)\, g^*_{\pi\,in}(k) - \left(\partial_\lambda \frac{g_{B\,in}(k)}{M}\right)\left(\partial^\lambda \frac{g^*_{\pi\,in}(k)}{M}\right)\right\}\Bigg] \\
& - \sum_l M^2 \Bigg[\left(\partial_\mu \frac{g_{\bar{c}\,in}(l)}{M}\right)\left(\partial_\nu \frac{g^*_{c\,in}(l)}{M}\right) + \left(\partial_\nu \frac{g_{\bar{c}\,in}(l)}{M}\right)\left(\partial_\mu \frac{g^*_{c\,in}(l)}{M}\right) \\
&\quad + g_{\mu\nu}\left\{ \alpha\, g_{\bar{c}\,in}(l)\, g^*_{c\,in}(l) - \left(\partial_\lambda \frac{g_{\bar{c}\,in}(l)}{M}\right)\left(\partial^\lambda \frac{g^*_{c\,in}(l)}{M}\right)\right\}\Bigg] \\
=\;& 0 \qquad (6.11)
\end{aligned}$$

by using (5.32) which is a necessary and sufficient condition for a BRST-invariant vacuum state.

B The effective equation of ϕ_c

The 1-loop effective equation of ϕ_c is given by

$$\begin{aligned}
\Box\phi_c \;+\;& \left(M^2+\xi R\right)\phi_c + \frac{\lambda}{2}\phi_c^3 - e^2\phi_c\langle U_\mu U^\mu\rangle + \frac{3}{2}\lambda\phi_c\langle\rho^2\rangle \\
+\;& \phi_c\left\langle \partial_\mu\frac{\breve{B}}{\phi_c}\partial^\mu\frac{\breve{B}}{\phi_c} - 2e^2\alpha\breve{B}^2\right\rangle + 2\phi_c\left\langle \partial_\mu\frac{\breve{B}}{\phi_c}\partial^\mu\frac{\pi}{\phi_c} - 2e^2\alpha\breve{B}\pi\right\rangle \\
+\;& 2i\phi_c\left\langle \partial_\mu\frac{\breve{\bar{c}}}{\phi_c}\partial^\mu\frac{\breve{c}}{\phi_c} - 2e^2\alpha\breve{\bar{c}}\breve{c}\right\rangle = 0 \qquad (6.12)
\end{aligned}$$

This can be rewritten as follows.

$$\begin{aligned}
\Box\phi_c \;+\;& \left(M^2+\xi R\right)\phi_c + \frac{\lambda}{2}\phi_c^3 - e^2\phi_c\langle U_\mu U^\mu\rangle + \frac{3}{2}\lambda\phi_c\langle\rho^2\rangle \\
-\;& 2i\phi_c\left\langle\left\{ i\boldsymbol{Q}_B,\; \partial_\mu\frac{\breve{\bar{c}}}{\phi_c}\partial^\mu\left(\frac{\pi}{\phi_c}+\frac{1}{2}\frac{\breve{B}}{\phi_c}\right) - 2e^2\alpha\breve{\bar{c}}\left(\pi+\frac{1}{2}\breve{B}\right)\right\}\right\rangle = 0 \qquad (6.13)
\end{aligned}$$

This shows again that there is no backreaction from the BRST quartet for a physical state in spite of their condensation.

VII Summary and Conclusion

We have investigated the U(1) Higgs model in spacetime-dependent background fields. In particular we choose a gauge fixing condition which is a generalization of the familiar R_ξ gauge.

The discrepancy between the in-vacuum and the out-vacuum leads to the condensation of physical and even unphysical particles. However, in case both vacuum states are physical, this can occur in a BRST-exact form. In other words, as is shown in [14], the BRST charge is invariant under the Bogoliubov transformation. The condensation of unphysical particles do not contribute to backreaction in the background field equations of a physical state because the corresponding terms are BRST-exact.

Appendix

1. Unitarity condition

$$\alpha_U {\alpha_U}^\dagger - \beta_U {\beta_U}^\dagger = \widetilde{I}, \qquad {\alpha_U}^T {\alpha_U}^* - {\beta_U}^\dagger \beta_U = \widetilde{I} \tag{A.1}$$

$$\alpha_U {\beta_U}^T - \beta_U {\alpha_U}^T = 0, \qquad {\alpha_U}^T {\beta_U}^* - {\beta_U}^\dagger \alpha_U = 0 \tag{A.2}$$

$$\alpha_{B\pi} {\alpha_{B\pi}}^\dagger - \beta_{B\pi} {\beta_{B\pi}}^\dagger = I, \qquad \bar{\alpha}^T_{B\pi} \bar{\alpha}^*_{B\pi} - \bar{\beta}^\dagger_{B\pi} \bar{\beta}_{B\pi} = I \tag{A.3}$$

$$\alpha_{B\pi} {\beta_{B\pi}}^T - \beta_{B\pi} {\alpha_{B\pi}}^T = 0, \qquad \bar{\alpha}^T_{B\pi} \bar{\beta}^*_{B\pi} - \bar{\beta}^\dagger_{B\pi} \bar{\alpha}_{B\pi} = 0 \tag{A.4}$$

$$\alpha_{B\pi} \bar{\alpha}^\dagger_{B\pi} - \beta_{B\pi} \bar{\beta}^\dagger_{B\pi} = I, \qquad \bar{\alpha}^T_{B\pi} {\alpha_{B\pi}}^* - \bar{\beta}^\dagger_{B\pi} \beta_{B\pi} = I \tag{A.5}$$

$$\alpha_{B\pi} \bar{\beta}^T_{B\pi} - \beta_{B\pi} \bar{\alpha}^T_{B\pi} = 0, \qquad \bar{\alpha}^T_{B\pi} {\beta_{B\pi}}^* - \bar{\beta}^\dagger_{B\pi} \alpha_{B\pi} = 0 \tag{A.6}$$

$$\alpha_{\bar{c}c} \bar{\alpha}^\dagger_{\bar{c}c} - \beta_{\bar{c}c} \bar{\beta}^\dagger_{\bar{c}c} = I, \qquad \bar{\alpha}^T_{\bar{c}c} {\alpha_{\bar{c}c}}^* - \bar{\beta}^\dagger_{\bar{c}c} \beta_{\bar{c}c} = I \tag{A.7}$$

$$\alpha_{\bar{c}c} \bar{\beta}^T_{\bar{c}c} - \beta_{\bar{c}c} \bar{\alpha}^T_{\bar{c}c} = 0, \qquad \bar{\alpha}^T_{\bar{c}c} {\beta_{\bar{c}c}}^* - \bar{\beta}^\dagger_{\bar{c}c} \alpha_{\bar{c}c} = 0 \tag{A.8}$$

$$\alpha_\rho {\alpha_\rho}^\dagger - \beta_\rho {\beta_\rho}^\dagger = I, \qquad {\alpha_\rho}^T {\alpha_\rho}^* - {\beta_\rho}^\dagger \beta_\rho = I \tag{A.9}$$

$$\alpha_\rho {\beta_\rho}^T - \beta_\rho {\alpha_\rho}^T = 0, \qquad {\alpha_\rho}^T {\beta_\rho}^* - {\beta_\rho}^\dagger \alpha_\rho = 0 \tag{A.10}$$

where

$$\widetilde{I}(ka, k'a') = \delta(ka, k'a') \tag{A.11}$$

$$I(k, k') = \delta(k, k') \tag{A.12}$$

$$(\alpha\alpha^\dagger)(k, k') = \sum_{k''} \alpha(k, k'')\alpha^*(k', k''), \quad \cdots \tag{A.13}$$

2. Symmetry of $\lambda(k, k')$

$$\begin{aligned} & \sum_{k''} \{\alpha(k, k'')\beta(k', k'') - \beta(k, k'')\alpha(k', k'')\} = 0 \\ \Longrightarrow \quad & \sum_{k''} \left\{\alpha^{-1}(k, k'')\beta(k'', k') - \alpha^{-1}(k', k'')\beta(k'', k)\right\} = 0 \\ \Longrightarrow \quad & \lambda(k, k') = \lambda(k', k) \end{aligned} \tag{A.14}$$

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